Optik - International Journal for Light and Electron Optics 171 (2018) 721–727

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Manipulating multi-frequency light in a five-level cascade EIT medium under Doppler broadening

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ARTICLE INFO

Keywords:
Group velocity
Electromagnetically induced transparency
Slowing light

ABSTRACT

An electromagnetically induced transparency (EIT) medium consisting of five-level-cascade atoms is proposed to control the group velocity of multi-frequency light under Doppler broadening. An analytic expression for the group index of probe light is derived as a function of the parameters of coupling light and temperature of the medium. It is shown that by adjusting intensity and/or frequency of the solely coupling light one may manipulate simultaneously group velocity of the probe light in three separated frequency regions, each of which enables to switch between the subluminal and superluminal modes. On the other hand, the effect of Doppler broadening increases the group velocity and enlarges the tuning range of the coupling intensity to switch between the superluminal and subluminal modes. The model is helpful to find applications related to the group velocity manipulation of multi-frequency light, and to serve future experimental studies at various temperature conditions.

1. Introduction

In the past few decades, slow (subluminal) and fast (superluminal) light effects with controllable transitions between superluminal and subluminal lights have potential impact on photonic technology which have opened up many important applications such as controllable optical delay lines, optical switching, telecommunication, interferometry, optical data storage and optical memories, and quantum information processing and so on [1–4]. In general, subluminal propagation takes place at the normal dispersive regions, while superluminal propagation normally corresponds to abnormal dispersive regions in a dispersive medium. An important aspect to manipulation of the light speed is controllable dispersive properties of a medium under laser fields.

The EIT is a quantum interference effect that makes a resonance medium becomes transparent and steeper dispersive for a probe light field under induction of other strong coupling light field [5]. Furthermore, magnitude and sign of dispersion of the medium for a probe light beam is controlled by another coupling light beam. Using EIT technique, several researchers attempted to demonstrate experimentally in subluminal [6–10] and superluminal [11–14] lights. Other studies have been done on the switching between the subluminal and superluminal propagations in an atomic medium by changing frequency, intensity, phase and polarization of applied fields [15–26].

Although, early studies on group velocity of light in three-level atomic systems have yielded breakthrough results and opened up many potential applications [1–18,20–22]. However, the three-level atomic configurations only create an EIT window in which light is controlled only in narrow frequency region. From practical perspective, extension from single to multi-window EIT is currently of

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https://doi.org/10.1016/j.ijleo.2018.06.148
Received 16 November 2017; Received in revised form 16 May 2018; Accepted 28 June 2018

interest due to its promising applications in multichannel optical communication, waveguides for optical signal processing and multichannel quantum information processing. A possible way for obtaining multi-window EIT is to use additionally controlling fields to excite various multi-level atomic systems [27,28]. Some research groups used such technique to control the group velocity at multiple frequencies [19,23,26,28].

A simplest way is to use only solely controlling field to couple simultaneously several closely-spaced hyperfine levels which was demonstrated for a five-level cascade scheme of cold ^{85}Rb atoms [29]. The observation was letter interpreted by an analytic model [30,31] which can be extended for related applications, *e.g.*, enhancement of Kerr nonlinearity [32,33], optical bistability (OB) [33], generating optical nano-fibers for guiding entangled beams [34], and EIT formation of laser pulse [35]. Specially, the analytical model is used to interpret the experimental observations of multi-window EIT spectrum [36]. Very recently, we have developed an analytical model to study influence of giant self-Kerr nonlinearity on group velocity of multi-frequency light [37]. The result showed that under self-Kerr nonlinearity, one may use the probe and/or coupling fields as knobs to manipulate the probe light between the subluminal and superluminal modes in three separated frequency regions. Furthermore, by using such a cascade excitation scheme, one may choose the uppermost excited electronic states having long lifetimes, as Rydberg states, to manipulate group velocity of light to a few mm/s.

So far, studies on control of group velocity of light in multi-window EIT media have often neglected the Doppler effect [19,23,26,27,37,38]. However, several related experiments working with gaseous medium often needed to carry out at normal conditions, which Doppler broadening should be taken into account. In a recent work [30], we studied absorption and dispersion spectra of the five-level atomic system under Doppler broadening by using an analytic method. The results showed that the slope of dispersion curve depends strongly on the temperature of medium. Growing of this interest, in this work, we study extensively on manipulation of the group velocity of a probe light in five-level cascade-type inhomogeneously broadened medium by using analytic method in a framework of density matrix theory. The influences of Doppler broadening and the parameters of coupling laser field on the group velocity of the probe light are investigated.

2. Theoretical model

We consider a five-level cascade-type atom system as shown in Fig. 1. A weak probe laser light (with frequency ω_p) drives the transition $|1\rangle \leftrightarrow |2\rangle$, whereas an intense coupling laser light (with frequency ω_c) couples simultaneously transitions between the state $|2\rangle$ and three closely-spacing states $|3\rangle$, $|4\rangle$ and $|5\rangle$. We denote δ_1 and δ_2 are frequency separations between the levels $|3\rangle$-$|4\rangle$ and

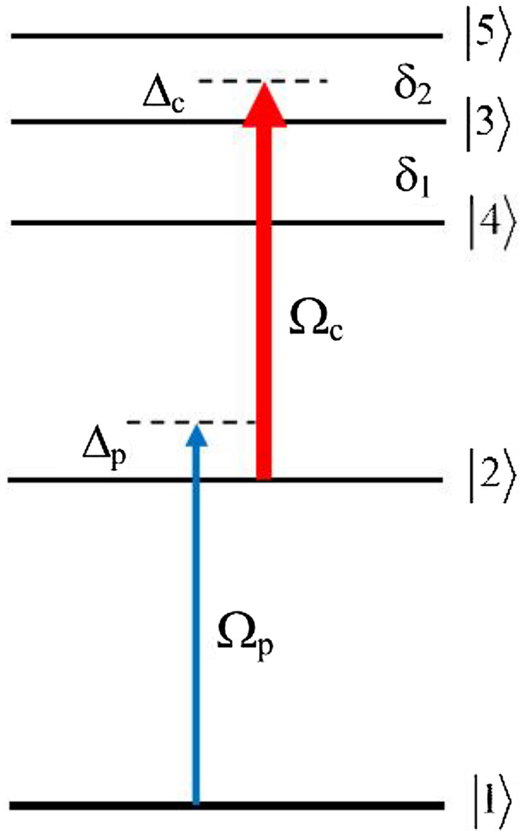

Fig. 1. The five-level cascade system.

|5⟩-|3⟩, respectively. The frequency detuning of the probe and coupling lights are respectively defined as:

$$\Delta_p = \omega_p - \omega_{21}, \quad \Delta_c = \omega_c - \omega_{32}. \tag{1}$$

The evolution of the system, which is represented via the density operator ρ, is represented by the following Liouville equation [30]:

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \Lambda\rho, \tag{2}$$

where, H and $\Lambda\rho$ represent the total Hamiltonian and relaxation operator, respectively.

Using the electric dipole- and rotating-wave approximations, the density matrix elements of the system are derived in the rotating frame as [30]:

$$\dot{\rho}_{51} = [i(\Delta_c + \Delta_p - \delta_2) - \gamma_{51}]\rho_{51} - \frac{i}{2}\Omega_p\rho_{52} + \frac{i}{2}\Omega_c a_{52}\rho_{21}, \tag{3}$$

$$\dot{\rho}_{41} = [i(\Delta_c + \Delta_p + \delta_1) - \gamma_{41}]\rho_{41} - \frac{i}{2}\Omega_p\rho_{42} + \frac{i}{2}\Omega_c a_{42}\rho_{21}, \tag{4}$$

$$\dot{\rho}_{31} = [i(\Delta_c + \Delta_p) - \gamma_{31}]\rho_{31} - \frac{i}{2}\Omega_p\rho_{32} + \frac{i}{2}\Omega_c a_{32}\rho_{21}, \tag{5}$$

$$\dot{\rho}_{21} = [i\Delta_p - \gamma_{21}]\rho_{21} - \frac{i}{2}\Omega_p(\rho_{22} - \rho_{11}) + \frac{i}{2}a_{32}\Omega_c\rho_{31} + \frac{i}{2}\Omega_c a_{42}\rho_{41} + \frac{i}{2}\Omega_c a_{52}\rho_{51}, \tag{6}$$

where, $\Omega_p = d_{21}E_p/\hbar$ and $\Omega_c = d_{32}E_c/\hbar$ are Rabi frequency of the probe and coupling light fields, respectively; d_{ik} is transition dipole moment between the $|i\rangle$ and $|k\rangle$ states; $a_{32} = d_{32}/d_{32}$, $a_{42} = d_{42}/d_{32}$, and $a_{52} = d_{52}/d_{32}$ are the relative transition strengths of the three transitions from the three hyperfine sublevels |3⟩, |4⟩, and |5⟩ to the level |2⟩; γ_{ik} represents the decay rate of the optical coherence ρ_{ik}, given by [30]:

$$\gamma_{ik} = \frac{1}{2}\left(\sum_{E_j<E_i}\Gamma_{ij} + \sum_{E_l<E_k}\Gamma_{kl}\right), \tag{7}$$

here, Γ_{ik} is the decay rate of the population from the level $|i\rangle$ to level $|k\rangle$.

In the weak field limit of the probe light and in the steady regime, the matrix element ρ_{21} is determined as [30]

$$\rho_{21} = \frac{\frac{i}{2}\Omega_p}{\gamma_{21} - i\Delta_p + \frac{a_{32}^2(\Omega_c/2)^2}{\gamma_{31} - i(\Delta_p + \Delta_c)} + \frac{a_{42}^2(\Omega_c/2)^2}{\gamma_{41} - i(\Delta_p + \Delta_c + \delta_1)} + \frac{a_{52}^2(\Omega_c/2)^2}{\gamma_{51} - i(\Delta_p + \Delta_c - \delta_2)}}. \tag{8}$$

The susceptibility of the atomic medium for the probe light relates to the matrix element ρ_{21} by

$$\chi = 2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21} = \frac{Nd_{21}^2}{\varepsilon_0\hbar}\left(\frac{A}{A^2 + B^2} + i\frac{B}{A^2 + B^2}\right), \tag{9}$$

where, A and B are real parameters:

$$A = -\Delta_p + \frac{A_{32}}{\gamma_{31}} + \frac{A_{42}}{\gamma_{42}} + \frac{A_{52}}{\gamma_{52}}, \tag{10}$$

$$B = \gamma_{21} + \frac{A_{32}}{\Delta_p + \Delta_c} + \frac{A_{42}}{\Delta_p + \Delta_c + \delta_1} + \frac{A_{52}}{\Delta_p + \Delta_c - \delta_2}, \tag{11}$$

and A_{ik} are effectively coupling parameters given by

$$A_{32} = \frac{\gamma_{31}(\Delta_p + \Delta_c)}{\gamma_{31}^2 + (\Delta_p + \Delta_c)^2}a_{32}^2\left(\frac{\Omega_c}{2}\right)^2, \tag{12}$$

$$A_{42} = \frac{\gamma_{41}(\Delta_p + \Delta_c + \delta_1)}{\gamma_{41}^2 + (\Delta_p + \Delta_c + \delta_1)^2}a_{42}^2\left(\frac{\Omega_c}{2}\right)^2, \tag{13}$$

$$A_{52} = \frac{\gamma_{51}(\Delta_p + \Delta_c - \delta_2)}{\gamma_{51}^2 + (\Delta_p + \Delta_c - \delta_2)^2}a_{52}^2\left(\frac{\Omega_c}{2}\right)^2. \tag{14}$$

For a vapor medium, it is necessary to take into account the Doppler broadening effect on the susceptibility. As in usual experiments for this configuration, the probe and coupling beams counter-propagate through the medium. Under this condition, an atom with velocity v moving towards the probe beam will therefore "see" a frequency up-shift (by $\omega_p v/c$) and down-shift (by $-\omega_c v/c$) for the probe and coupling lights, respectively. In such Doppler shifts, the frequency detunings are therefore modified to

$$\Delta_p = \omega_p - \omega_{21} + \omega_p v/c\,;\ \Delta_c = \omega_c - \omega_{32} - \omega_c v/c. \quad (15)$$

In a thermal equilibrium condition at temperature T, the number of particles having velocity v is assumed to obey the following Maxwell-Boltzmann distribution

$$N(v) = \frac{N_0}{u\sqrt{\pi}} e^{-v^2/u^2} dv, \quad (16)$$

where,

$$u = \sqrt{\frac{2k_B T}{m}}, \quad (17)$$

is the favorable velocity, m is mass of the particle; N_0 is the total number of particles occupied in a volume unit.

Under the above assumptions of the Doppler effect, the susceptibility in Eq. (9) is integrated over the velocity from $-\infty$ to $+\infty$, given by

$$\chi = \int_{-\infty}^{+\infty} \chi(v) dv = \frac{iN_0 d_{21}^2 \sqrt{\pi}}{\varepsilon_0 \hbar \left(\frac{\omega_p u}{c}\right)} e^{z^2}[1 - erf(z)], \quad (18)$$

where, $erf(z)$ is the error function with a complex argument z:

$$z = \frac{c}{\omega_p u}(B + iA), \quad (19)$$

with, A and B are the parameters defined by Eqs. (10) and (11), respectively. The expression (18) represents the susceptibility for the probe light as a function of controllable parameters of the coupling light and temperature of the medium.

According to the Kramers–Kronig relation, the absorption and dispersion coefficients represent respectively to imaginary χ'' and real χ' parts of the susceptibility $\chi = \chi' + i\chi''$. The absorption and dispersion coefficients for the probe light can therefore be determined via the following relations:

$$\alpha = \frac{\omega_p \chi''}{c}, \quad (20)$$

$$n = 1 + \frac{\chi'}{2}. \quad (21)$$

The group index for the probe field in the case of Doppler broadening can be written as

$$n_g^{(D)} = n + \omega_p \frac{\partial n}{\partial \omega_p}. \quad (22)$$

The group velocity and the group index are related as

$$v_g^{(D)} = \frac{c}{n_g^{(D)}}. \quad (23)$$

In the case of Doppler absented, the linear index n_0 and the group index $n_g^{(0)}$ are determined by:

$$n^{(0)} = 1 + \frac{Nd_{21}^2}{2\varepsilon_0 \hbar} \frac{A}{A^2 + B^2}, \quad (24)$$

$$n_g^{(0)} \simeq \omega_p \frac{Nd_{21}^2}{2\varepsilon_0 \hbar}\left[\frac{A'(A^2 + B^2) - 2A(AA' + BB')}{(A^2 + B^2)^2}\right], \quad (25)$$

where, A' and B' represent the derivatives of A and B over ω_p, respectively, which are given by:

$$A' = -1 + \frac{A_{32}}{\gamma_{31}(\Delta_p + \Delta_c)} - \frac{2A_{32}^2}{a_{32}^2(\Omega_c/2)^2\gamma_{31}^2} + \frac{A_{42}}{\gamma_{41}(\Delta_p + \Delta_c + \delta_1)} - \frac{2A_{42}^2}{a_{42}^2(\Omega_c/2)^2\gamma_{41}^2} + \frac{A_{52}}{\gamma_{51}(\Delta_p + \Delta_c - \delta_2)} - \frac{2A_{52}^2}{a_{52}^2(\Omega_c/2)^2\gamma_{51}^2}, \quad (26)$$

$$B' = -\frac{2A_{32}^2}{a_{32}^2(\Omega_c/2)^2\gamma_{31}(\Delta_p + \Delta_c)} - \frac{2A_{42}^2}{a_{42}^2(\Omega_c/2)^2\gamma_{41}(\Delta_p + \Delta_c + \delta_1)} - \frac{2A_{52}^2}{a_{52}^2(\Omega_c/2)^2\gamma_{51}(\Delta_p + \Delta_c - \delta_2)}. \quad (27)$$

3. Results and discussion

As an illustration of the analytic results, we apply to the ^{85}Rb atomic vapor. The states, $|1\rangle$, $|2\rangle$, $|3\rangle$, $|4\rangle$ and $|5\rangle$ are chosen as $5S_{1/2}(F = 3)$, $5P_{3/2}(F' = 3)$, $5D_{5/2}(F'' = 3)$, $5D_{5/2}(F'' = 4)$, and $5D_{5/2}(F'' = 3)$, respectively. The atomic parameters are given by [31,38]: $N = 5 \times 10^{11}$ atoms/cm^3; $\Gamma_{32} = \Gamma_{42} = \Gamma_{52} = 2\pi \times 0.97$ MHz; $\Gamma_{21} = 2\pi \times 6$ MHz; $\delta_1 = 2\pi \times 9$ MHz; $\delta_2 = 2\pi \times 7.6$ MHz; $d_{21} = 1.6 \times 10^{-29}$ C.m; $\omega_p = 2\pi \times 3.77 \times 10^8$ MHz and a_{32}: a_{42}: a_{52} = 1:1.4:0.6.

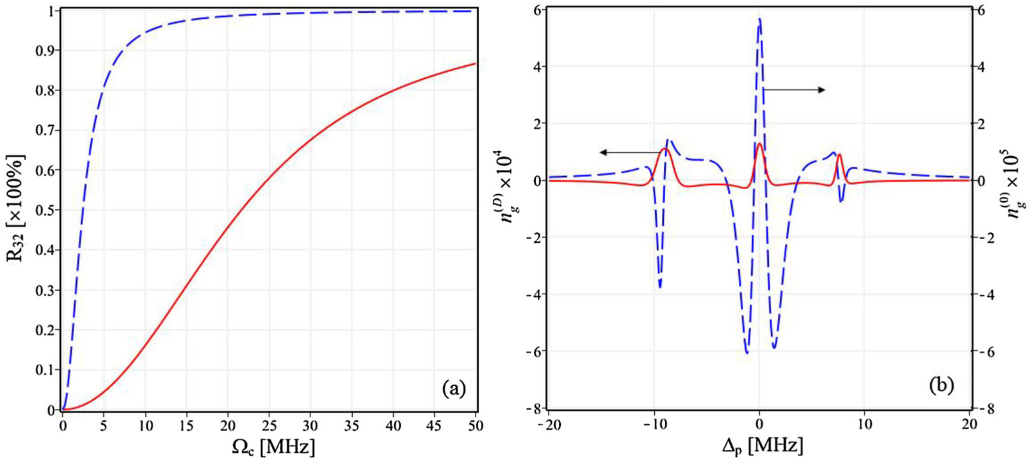

Fig. 2. Plot of transparent efficiency R_{32} (a) and group index (b) for the cases of the Doppler broadening absented (dashed line) and presented (solid line) when $\Delta_c = 0$; In the Fig. 2(b), the value of Ω_c is chosen as 2.5 MHz for the absence and 22 MHz for the presence of Doppler broadening, respectively.

In a first step, we consider the effect of Doppler broadening by plotting the group index versus the probe frequency detuning for the cases of absence (dashed line) and presence (solid line) of Doppler broadening (at room temperature $T = 300$ K), as shown in Fig. 2. As pointed in [37] that the maximum of group index can be attained at the transparency efficiency about 50%, therefore to compare the group velocity in both case we employed the intensity of the coupling laser Ω_c at the values that correspond to transparent efficiency of 50% for each case. Fig. 2(a) shows the variation of transparent efficiency R_{32} with the respect to Ω_c. It is shown that the transparent efficiency approaches to 50% when $\Omega_c = 2.5$ MHz or $\Omega_c = 22$ MHz which correspond to the Doppler effect absented or presented (at $T = 300$ K), respectively.

As shown in from Fig. 2(b), there exists three frequency regions of the probe light (centered at $\Delta_p = -9$ MHz, $\Delta_p = 0$, and $\Delta_p = 7.6$ MHz) each of which behavior positive and negative group index, or subluminal and superluminal propagation, respectively. On the other hand, Doppler broadening leads to decrease of the group index or increase of the group velocity by few orders compared to the case of Doppler absented. For example, at $\Delta_p = 0$ as in Fig. 2(b), $n_g^{(0)} \sim 5.6 \times 10^5$ (or group velocity $v_g^{(0)} \sim 5.3 \times 10^2$ m/s), while $n_g^{(0)} \sim 1.2 \times 10^3$ (or group velocity $v_g^{(D)} \sim 2.5 \times 10^5$ m/s). In order to describe further the influence of Doppler broadening, we plotted the group index versus the temperature at a fixed value $\Omega_c = 22$ MHz, that corresponds to 50% of the R_{32}, as displayed in Fig. 3. It is found that as the temperature increases the magnitude of group index decreases gradually.

In order to see a control of location of subluminal and superluminal regions with the coupling field's frequency we plotted the group index $n_g^{(D)}$ versus Δ_p at different values $\Delta_c = -5$ MHz, $\Delta_c = 0$, and $\Delta_c = 5$ MHz as illustrated in Fig. 4. It is shown in Fig. 4(a) that one may shift systematically the subluminal and superluminal regions to the red or blue directions according to the coupling frequency tuned to the blue or red side, respectively. On the other hand, as shown in Fig. 4(b), the subluminal light can be switched into a superluminal light, and vice versa, with variation of the coupling field's frequency.

In Fig. 5, we plotted the group index $n_g^{(D)}$ (solid line) versus the coupling Rabi frequency (intensity) Ω_c at the fixed values

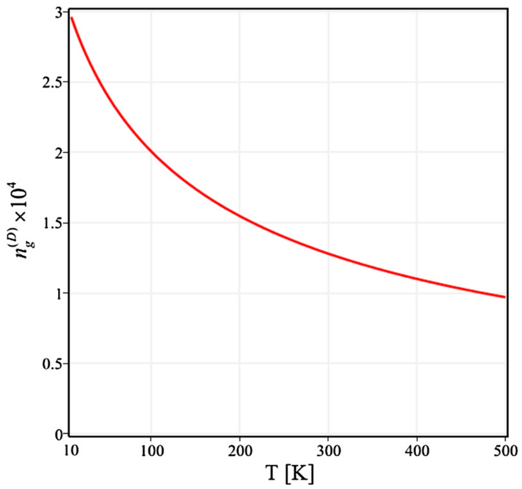

Fig. 3. Variation of group index $n_g^{(D)}$ versus the temperature at $\Delta_p = \Delta_c = 0$ and $\Omega_c = 22$ MHz.

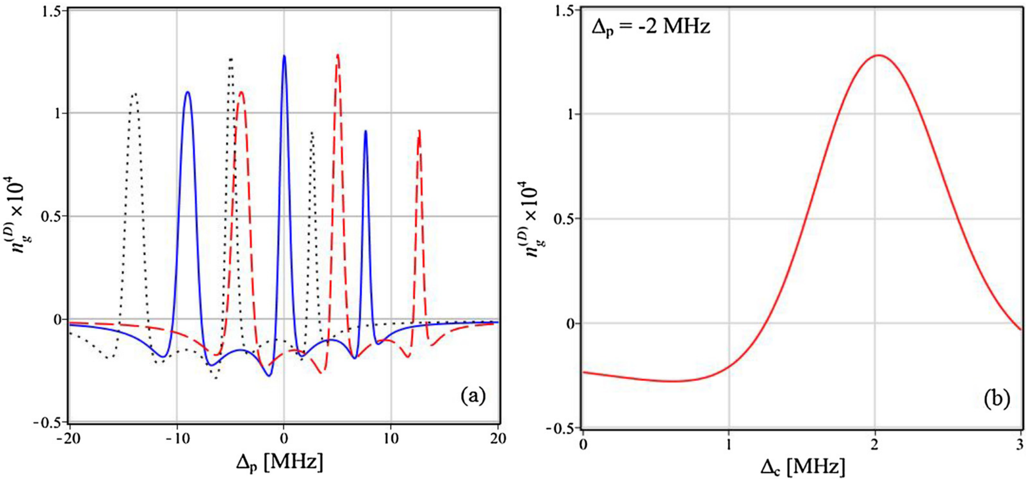

Fig. 4. (a) –Systematic shift of the subluminal and superluminal regions with tuning frequency $\Delta_c = -5$ MHz (*dashed*), $\Delta_c = 0$ (*solid*), and $\Delta_c = 5$ MHz (*dotted*); (b) – Variation of the group index $n_g^{(D)}$ versus Δ_c when $\Delta_p = -2$ MHz. Both cases are plotted at $\Omega_c = 22$ MHz and $T = 300$ K. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

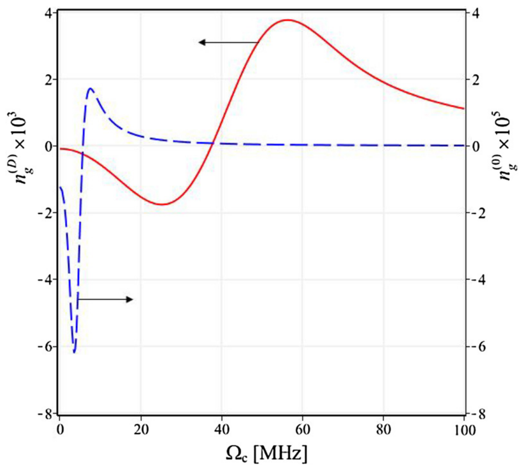

Fig. 5. Plots of $n_g^{(0)}$ (dashed line) and $n_g^{(D)}$ (solid line) versus Ω_c when $\Delta_p = 2$ MHz, $\Delta_c = 0$, and $T = 300$ K.

$\Delta_p = 2$ MHz, $\Delta_c = 0$ and $T = 300$ K. It is seen that, with variation of the coupling intensity, the group index is varied between negative and positive values. As a consequence, the probe light propagation can be manipulated between the superluminal and subluminal modes. On the other hand, the tuned range of coupling Rabi frequency Ω_c needed to switch between two adjacent extreme values of the group index in each EIT window is bigger in the case of Doppler broadening (see dashed line in Fig. 5(b)). Indeed, tuning range of Ω_c to switch from a negative peak to a nearby positive peak of the group index is about $\Delta\Omega_c = 4$ MHz or $\Delta\Omega_c = 30$ MHz for the absence or presence of Doppler broadening, respectively. This difference can be explained with a notice that the Doppler broadening makes absorption profile broader thus enlarges distance between two adjacent positive and negative peaks of the group index.

4. Conclusion

We have proposed a five-level-cascade atomic medium to manipulate group velocity of multiple frequency probe light under Doppler broadening. An analytic expression for the group index has been derived as a function of the parameters of a solely coupling light and temperature of the medium. It is shown that one may tune intensity and/or frequency of the coupling light field control group velocity of the probe light in three separated frequency regions, each of which enables to manipulate between the subluminal and superluminal modes. On the other hand, Doppler broadening enhances group velocity and enlarges the needed tuning range of the coupling intensity to switch between the superluminal and subluminal modes. The analytic form and the model of controllable multi-frequency light can find related applications and helpful for experimental studies at various temperature conditions.

Acknowledgment

The financial support from Vietnamese Ministry of Science and Technology under the grant code ĐTĐLCN.17/17 is acknowledged.

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